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**Question 1: N^2 < 2^N when N ≥ 5**

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| --- | --- | --- |
| **N** | **N^2** | **2^N** |
| 1 | 1 | 2 |
| 2 | 4 | 4 |
| 3 | 9 | 8 |
| 4 | 16 | 16 |
| 5 | 25 | 32 |
| 6 | 36 | 64 |
| 7 | 49 | 128 |
| 8 | 64 | 256 |

For K = 5

N^2 < 2^N

25 < 32

Assuming for K = N (N > 5) is true

Check K = N + 1

(N + 1)^2 < 2^(N + 1)

N^2 + 2N + 1 < 2\*2^N

Knowing N^2 < 2^N, can simply evaluate 2N + 1 < 2^N which is true for all N > 5

**Question 2: 2^N < N! when N ≥ 4**

|  |  |  |
| --- | --- | --- |
| **N** | **2^N** | **N!** |
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 3 | 8 | 6 |
| 4 | 16 | 24 |
| 5 | 32 | 120 |
| 6 | 64 | 720 |
| 7 | 128 | 5040 |
| 8 | 256 | 40320 |

For K = 4

2^N < N!

16 < 24

Assuming true for K = N (N > 4) is true

Checking K = N + 1

2^(N + 1) < (N + 1)!

2(2^N) < (N!)(N+1)

knowing 2^N < N!, can simply evaluate 2 < N + 1 which is true for all N > 4

**Question 4: Dominoes**

D(n) = D(n-1) + D(n-2) for n > 2

|  |  |  |  |
| --- | --- | --- | --- |
| **N** | **D(N)** | **2^N** | **(3/2)^N** |
| 1 | 1 | 2 | 1.5 |
| 2 | 2 | 4 | 2.25 |
| 3 | 3 | 8 | 3.375 |
| 4 | 5 | 16 | 5.0625 |
| 5 | 8 | 32 | 7.59375 |
| 6 | 13 | 64 | 11.390625 |
| 7 | 21 | 128 | 17.0859375 |
| 8 | 34 | 256 | 25.62890625 |

Taking the previous cases and adding a single vertical domino gives the D(n-1) case

Taking the previous of the previous case and adding vertical dominos or a set of horizontal dominoes gives the D(n-2) case

Summing these two cases gives you the permutations of the D(n) case

Prove k=3 is true

D(3) = D(2) + D(1)

D(3) = 2 + 1

Assuming k=n is true

Prove k=n+1 is true

D(n+1) = D(n) + D(n-1)

- b/c D(n) is true, D(n-1) is true, thus D(n+1) is true

2^n > D(n) > (3/2)^n

Prove k=5 is true

32 > 8 > 7.59…

Assuming k=n is true

Prove k=n+1 is true

2^(n+1) > D(n+1) > (3/2)^(n+1)

2(2^n) > D(n) + D(n-1) > (3/2)(3/2)^n

- b/c we assume 2^n > D(n) > (3/2)^n is true,

- we just need to evaluate 2 > 1 + D(n-1)/D(n) > (3/2)

- for values of n > 5, the ratio of D(n-1)/D(n) is always > 0.5 thus the k=n+1 case is true